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ON THE TRANSCENDENTAL EQUATION WITH THREE UNKNOWNS $2(x+y)-3\sqrt{xy}=(k^2+7s^2)z^2$

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ABSTRACT
The transcendental equation with three unknowns given by $2(x+y)-3\sqrt{xy}=(k^2+7s^2)z^2$ is considered and analyzed for finding different sets of integer solutions.

KEYWORDS: Transcendental equation, Integer solutions.

1. INTRODUCTION
The subject of diophantine equation, one of the interesting areas of Number Theory, plays a significat role in higher arithmetic and has a marvelous effect on credulous people and always occupies a remarkable position due to unquestioned historical importance. The diophantine equations may be either polynomial equation with atleast two unknowns for which integer solution, are required or transcendental equation involving trigonometric, logarithmic, exponential and surd function such that one may be interested in getting integer solution.

It seems that much work has not been done with regard to integer solution for transcendental equation with surds. In this context, one may refer [1-10].

In this paper, we are interested in obtaining integer solutions to transcendental equation involving surds. In particular, we obtain different sets of integer solutions to the transcendental equation with three unknowns given by $2(x+y)-3\sqrt{xy}=(k^2+7s^2)z^2$.

2. METHOD OF ANALYSIS
The ternary transcendental equation to be solved is

$$2(x+y)-3\sqrt{xy}=(k^2+7s^2)z^2 \tag{1}$$

Introduction of the transformations

$$x=(u+v)^2;\ y=(u-v)^2;\quad u\neq v\neq 0 \tag{2}$$

in (1) leads to

$$u^2+7v^2=(k^2+7s^2)z^2 \tag{3}$$

The above equation (3) is solved through different methods and using (2), one obtains different sets of solutions to (1).

2.1 Method 1:
Assume

$$z = a^2 + 7b^2 \quad (4)$$

Substituting (4) in (3) and applying the method of factorization, consider

$$u + i\sqrt{7}v = (k + i\sqrt{7}s)(a + i\sqrt{7}b)^2$$

On equating the rational and irrational parts, one obtains

$$u = k(a^2 - 7b^2) - 14sab$$
$$v = s(a^2 - 7b^2) + 2kab$$

In view of (2), we have

$$\left.\begin{aligned} x &= [k(a^2 - 7b^2 + 2ab) + s(a^2 - 7b^2 - 14ab)]^2 \\ y &= [k(a^2 - 7b^2 - 2ab) - s(a^2 - 7b^2 + 14ab)]^2 \end{aligned}\right\} \quad (5)$$

Thus, (4) and (5) represent the integer solutions to (1).

2.2 Method 2:
Write (3) in the form of ratio as

$$\frac{u + kz}{sz + v} = \frac{7(sz - v)}{u - kz} = \frac{\alpha}{\beta}, \beta \neq 0$$

which is equivalent to the system of double equations

$$\beta u - \alpha v + (k\beta - s\alpha)z = 0$$
$$\alpha u + 7\beta v - (k\alpha + 7s\beta)z = 0$$

Applying the method of cross multiplication and using (2), the values of x, y and z satisfying (1) are given by

$$x = [k(\alpha^2 - 7\beta^2 + 2\alpha\beta) - s(\alpha^2 - 7\beta^2 - 14\alpha\beta)]^2$$
$$y = [k(\alpha^2 - 7\beta^2 - 2\alpha\beta) + s(\alpha^2 - 7\beta^2 + 14\alpha\beta)]^2$$
$$z = 7\beta^2 + \alpha^2$$

2.3 Method 3:
Equation (3) is written as

$$u^2 + 7v^2 = (k^2 + 7s^2)z^2 * 1 \quad (6)$$

Assume

$$1 = \frac{(3 + i\sqrt{7})(3 - i\sqrt{7})}{16} \quad (7)$$

Substituting (4) and (7) in (6) and factorizing, consider

$$u + i\sqrt{7}v = (k + i\sqrt{7}s)\frac{(3 + i\sqrt{7})}{4}(a + i\sqrt{7}b)^2$$

On equating the real and imaginary parts, we have

$$u = \frac{1}{4}[(a^2 - 7b^2)(3k - 7s) - 14ab(3s + k)]$$
$$v = \frac{1}{4}[(a^2 - 7b^2)(k + 3s) + 2ab(3k - 7s)]$$

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Replacing a by 2A and b by 2B, in the above values of u, v and (4), the corresponding integer solutions to (1) are represented by

$$x = [4k(A^2 - 7B^2 - 2AB) - 4s(A^2 - 7B^2 + 14AB)]^2$$
$$y = [2k(A^2 - 7B^2 - 10AB) - 2s(5A^2 - 35B^2 + 14AB)]^2$$
$$z = 4(A^2 + 7B^2)$$

Note:

In (7), representing 1 as

$$1 = \frac{(1 + i3\sqrt{7})(1 - i3\sqrt{7})}{64}$$

another set of solutions to (1) are obtained.

2.4 Method 4:

Considering

$$x = K^2 y \qquad (8)$$

in (1), it gives

$$(2K^2 - 3K + 2)y = (k^2 + 7s^2)z^2$$

which is satisfied by

$$y = (k^2 + 7s^2)(2K^2 - 3K + 2)\alpha^2 \qquad (9)$$
$$z = (2K^2 - 3K + 2)\alpha \qquad (10)$$

In view of (8), we have

$$x = (k^2 + 7s^2)K^2(2K^2 - 3K + 2)\alpha^2 \qquad (11)$$

Note that (9), (10) and (11) satisfy (1).

2.5 Method 5:

Introduction of the transformations

$$x = 2R^2,\ y = 2S^2$$

in (1) leads to

$$4(R^2 + S^2) - 6RS = (k^2 + 7s^2)z^2 \qquad (12)$$

Treating (12) as a quadratic in R and solving for R, we get

$$R = \frac{1}{4}\left[3S \pm \sqrt{4(k^2 + 7s^2)z^2 - 7S^2}\right] \qquad (13)$$

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The square root on the RHS of (13) is eliminated when

$$z = 7Q^2 + P^2 \tag{14}$$

$$S = 4kPQ - 2sP^2 + 14Q^2 s \tag{15}$$

Using the above values of z and S in (13), we have

$$\left.\begin{aligned} R &= \frac{1}{4}[PQ(12k+28s) + P^2(2k-6s) + Q^2(42s-14k)] \\ R &= \frac{1}{4}[PQ(12k-28s) - P^2(2k+6s) + Q^2(42s+14k)] \end{aligned}\right\} \tag{16}$$

Replacing P by 2M and Q by 2N in (14), (15) and (16) the corresponding integer solutions to (1) are obtained as below:

$$x = 2[MN(12k+28s) + M^2(2k-6s) + N^2(42s-14k)]^2$$
$$x = 2[MN(12k-28s) - M^2(2k+6s) + N^2(42s+14k)]^2$$
$$y = 2[16kMN - 8sM^2 + 56sN^2]^2$$
$$z = 4(7M^2 + N^2)$$

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